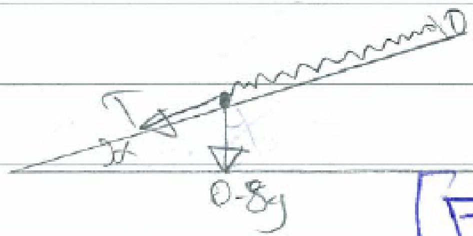
M3 - June 2005

1 –



$$T = \frac{\lambda x}{a} = \frac{20 \times 0.4}{2} = 4N$$

$[F = ma]$

$$4 + 0.8g\sin\alpha = 0.8a$$

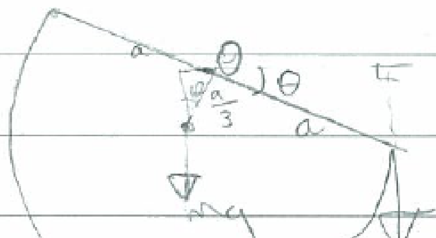
$$a = \frac{4 + 0.8 \times 9.8 \times \frac{3}{5}}{0.8} = 10.88\ ms^{-2}$$

2 – a) $\frac{1}{2}r \cdot 2\pi r^2 = (2\pi r^2 + \pi r^2)\bar{d}$

$$a = 3\bar{d}$$

$$\bar{d} = \frac{1}{3}a$$

b)

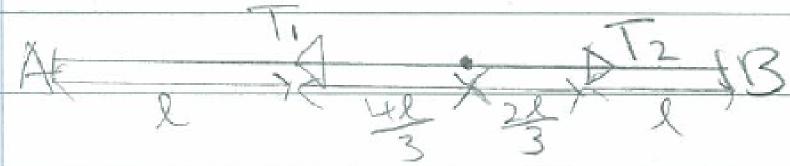


$$\frac{1}{2}Mga\cos\theta = \frac{Mga\sin\theta}{3}$$

$$3 \cdot \frac{1}{2} = \tan\theta$$

$\theta = 56°\ (2sf)$

3.



a) $\cancel{T_1 = \frac{\lambda x}{a} = \frac{4mg \cdot 4l}{3l} = \frac{16mg}{3}}$ $T_1 = \frac{4mg(l+x)}{l}$

$\cancel{T_2 = \frac{\lambda x}{a} = \frac{4mg \cdot 2l}{3l} = \frac{8mg}{3}}$ $T_2 = \frac{4mg(l-x)}{l}$

$\frac{4\cancel{m}g(l-x)}{l} - \frac{4\cancel{m}g(l+x)}{l} = \cancel{m}\ddot{x}$

$4g - \frac{4gx}{l} - 4g - \frac{4gx}{l} = \ddot{x}$

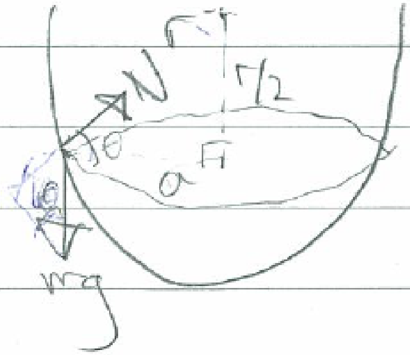
$-\frac{8gx}{l} = \ddot{x}$

$\therefore$ SHM with $\omega^2 = \frac{8g}{l}$

c) In SHM, max vel occurs at the centre of oscillation

$V = \omega r = \sqrt{\frac{8g}{l}} \times \frac{l}{3} = \frac{2l\sqrt{2g}}{3\sqrt{l}} = \frac{2\sqrt{2gl}}{3}$

4.



a) ~~$N = mg\sin\theta = mg\frac{\sqrt{3}}{2}r$~~

$mg = N\sin\theta$

$N = \frac{mg}{\sin\theta} = \frac{mg}{\frac{r}{2}\times\frac{1}{r}} = 2mg$

b) $\boxed{F = ma} \rightarrow$

$N\cos\theta = m\omega^2 a$

$N\cos\theta = m\omega^2 r\cos\theta$

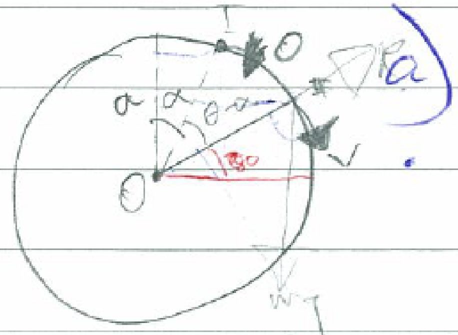
$2mg = m\omega^2 r$

$\omega^2 = \frac{2g}{r}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{2g}{r}}}$

$= \frac{2\pi}{\sqrt{2g}} \times \sqrt{r} = \frac{\pi\sqrt{2gr}}{g}$

5.



a) $mE_t = mE_T$

$mg(a\cos\alpha - a\cos\theta) = \frac{1}{2}mv^2$

$v^2 = 2ga(\cos\alpha - \cos\theta)$

b) $\boxed{F = ma}$

$$mg\cos\theta - R = \frac{m}{a}\left(2ga\cos\alpha - 2ga\cos\theta\right)$$

$$\cancel{m}g\cos\theta = 2g\cancel{a}\cdot\frac{3}{4_2} - 2g\cancel{a}\cos\theta$$

$$3g\cos\theta = \frac{3}{2}g$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60^\circ$$

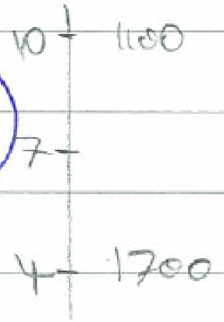
c) $v^2 = 2ga\cdot\frac{3}{4_2} - 2ga\cdot\frac{1}{2} = \frac{3}{2}ga - ga = \frac{1}{2}ga$

$$\frac{1}{2}\cancel{m}\cdot\frac{1}{2}ga + \cancel{m}g\cdot\left(a + \frac{a}{2}\right) = \frac{1}{2}\cancel{m}V^2$$

$$\frac{1}{4}ga + \frac{3}{2}ga = \frac{1}{2}V^2$$

$$V^2 = \frac{7}{2}ga \qquad V = \sqrt{\frac{7}{2}ga}$$

6. a)



$a = 3\,\text{m}$ $\qquad w = \frac{2\pi}{T} = \frac{\pi}{6}$ rad h^{-1}

$T = 12\,\text{h}$

$x = a\cos wt$

$x = 3\cos\frac{\pi}{6}\cdot 5 = 3\cos\frac{5\pi}{6} = -\frac{3\sqrt{3}}{2}$

$v^2 = w^2(a^2 - x^2)$

$v^2 = \frac{\pi^2}{36}\left(9 - \frac{27}{4}\right) = \frac{\pi^2}{16}$

$v = \frac{\pi}{4}\,\text{m}\,\cancel{s}h^{-1}$

b) $-1.5 = 3\cos\frac{\pi}{6}t$

$-\frac{1}{2} = \cos\frac{\pi}{6}t$

$\frac{4\pi}{3}, \frac{2\pi}{3} = \frac{\pi}{6}t$

$t = 4, 8 \quad \therefore$ Total time is 4h

7 – a) $\boxed{F = ma}$

$$-\frac{k}{(x+1)^2} = \frac{1}{3} v \frac{dv}{dx}$$

$$-k\int_1^8 (x+1)^{-2}\,dx = \frac{1}{3}\int_4^{\sqrt{2}} v\,dv$$

$$+k\left[(x+1)^{-1}\right]_1^8 = \frac{1}{3}\cdot\frac{1}{2}\left[v^2\right]_4^{\sqrt{2}}$$

$$k\left(\frac{1}{9} - \frac{1}{2}\right) = \frac{1}{6}(2 - 16)$$

$$+\frac{7}{18}k = +\frac{7}{3}$$

$$k = \frac{18 \cdot 7}{3 \cdot 7} = 6$$

b) $$-6\int_1^x (x+1)^{-2}\,dx = \frac{1}{3}\int_4^0 v\,dv$$

$$6\left[(x+1)^{-1}\right]_1^x = \frac{1}{6}\left[v^2\right]_4^0$$

$$\frac{6}{x+1} - 3 = -\frac{8}{3}$$

$$6 = \frac{x+1}{3}$$

$$x + 1 = 18$$

$$x = 17\text{m}$$